In the first analytical model an analysis was accomplished on a isotropic thick-walled cylinder under tensile circumferential stress. Derivation of the equations of equilibrium in the stress form were developed which were similar to those disclosed in S.P. Timoshenko and J.N. Goodier, Theory of Elasticity, 3 rd edition, McGraw-Hill, 1970. Originally published in 1934. p 65- 69, which is incorporated herein by reference. Computation results were presented for 10 and 50 layers of the tube showing the stress distribution in the roll. Equations relating stresses and strains were also presented for plane stress and plane strain models. Orthotropic and isotropic materials were considered. Computation results were also presented for the strain distribution, and the distribution of EFL.

The problem of stress distribution in a roll can be considered as an axisymmetric problem in the polar coordinate system. A typical model element of a thick walled cylinder 10, shown in a polar coordinate system, is shown in Figures 4A, 4B and 4C. In these Figures a element "E" of a thick-walled cylinder 10 is shown, where the element E has sides 11, 12, 13 and 14.

Timoshenko (1970) presented equations of equilibrium in the polar system of coordinates based on the equilibrium of a small element. This element is cut out of a ring or a cylinder by the radial sections normal to the plane of Figure 4A and is shown by bold lines 11, 12, 13 and 14, in Figure 4B. The normal stress in the circumferential direction is denoted by σ_Θ, and the stress in the radial direction as σ_r. Components of shear stress are denoted by $\tau_{r\Theta}$. The radial force on the right side 11 of the element is equal to (σ_r r dΘ) and radial force on the left side 13 of the element is equal to (-σ_r r dΘ). The normal forces on the upper and

lower perpendicular sides 14 and 12 are correspondingly ($-\sigma_\Theta$ dr dΘ/2) and (σ_Θ dr dΘ/2). The shearing forces on the upper and lower sides are $[(\tau_{r\Theta})_{12} - (\tau_{r\Theta})_{14}]$ dr.

Summing up forces in the radial direction, including the body force R per unit volume in the radial direction, produces the following equation:

$(\sigma_r\, r)_{11}\, d\Theta - (\sigma_r\, r)_{13}\, d\Theta - (\sigma_\Theta)_{12}\, dr\, d\Theta/2 - (\sigma_\Theta)_{14}\, dr\, d\Theta/2 + [(\tau_{r\Theta})_{12} - (\tau_{r\Theta})_{14}]\, dr + R\, r\, dr\, d\Theta = 0;$

Dividing by (dr dΘ) results in:

$$\frac{(\sigma_r r)_{11} - (\sigma_r r)_{13}}{dr} - \frac{1}{2}\left[(\sigma_\Theta)_{12} + (\sigma_\Theta)_{14}\right] + \frac{(\tau_{r\Theta})_{12} - (\tau_{r\Theta})_{14}}{d\Theta} + Rr = 0.$$

When the element dimensions approach infinitesimally small values, the first and third term of this equation represent the first derivatives, while the second term is an average value of σ_Θ. The equation of equilibrium in the tangential direction can be derived in the same manner. The two equations take the following final form:

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\Theta}}{\partial \Theta} + \frac{\sigma_r - \sigma_\Theta}{r} + R = 0 \qquad (3.1)$$

$$\frac{1}{r}\frac{\partial \sigma_\Theta}{\partial \Theta} + \frac{\partial \tau_{r\Theta}}{\partial r} + \frac{2\tau_{r\Theta}}{r} + S = 0, \qquad (3.2)$$

where S is the component of body force (per unit volume) in the tangential direction. When the body forces are equal to zero, equations 3.1 and 3.2 can be solved using the stress function, Φ, that generally depends on radial coordinate, r, and angular coordinate, Θ.

$$\sigma_\theta = \frac{\partial^2 \Phi}{\partial r^2}; \qquad (3.3)$$

$$\sigma_r = \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} \qquad (3.4)$$

$$\tau_{r\theta} = \frac{1}{r}\frac{\partial \Phi}{\partial \theta} - \frac{1}{r}\frac{\partial^2 \Phi}{\partial r\, \partial \theta} \,. \tag{3.5}$$

The stress distribution in a roll can be considered as a function of radius only. Applying the condition of independence of the stress function from angular coordinate, Θ, Equations 3.3 – 3.5 will change as follows:

$$\sigma_\theta = \frac{\partial^2 \Phi}{\partial r^2}; \tag{3.6}$$

$$\sigma_r = \frac{1}{r}\frac{\partial \Phi}{\partial r}; \tag{3.7}$$

$$\tau_{r\theta} = 0. \tag{3.8}$$

The form of stress function for this type of problem is suggested in Timoshenko (1970) to be:

$$\Phi = A\ Ln\ r + B\ r^2 Ln\ r + C\ r^2 + D\,, \tag{3.9}$$

where A, B, and C are unknown constants.

Substitution of Equation 3.9 into Equations 3.6 and 3.7 results in the following expressions for the stresses:

$$\sigma_\theta = -\frac{A}{r^2} + B\ (3 + 2Ln\ r) + 2C; \tag{3.10}$$

$$\sigma_r = \frac{A}{r^2} + B\ (1 + 2Ln\ r) + 2C. \tag{3.11}$$

In order to model stress conditions typical for reeled buffer tubes, the boundary conditions shown in Figure 4C were considered. Three unknown constants can be found from the following three boundary conditions: